The Optimal Adjustment of Bank Capital Regulation in a Downturn

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May 5, 2016

Abstract

The procyclicality of bank capital regulation has become a key concern. This paper analyzes how capital requirements should adjust to economic shocks especially during a downturn. Whenever bank capital is scarce, optimal regulation trades off the low risk of costly bank failure against the investment capacity of entrepreneurs. Adding a full-fledged model of the loan market reveals important equilibrium effects as changes in the state of the economy affect optimal capital requirements through the lending rate and the optimal risk level. The adjustment fundamentally differs between two shocks: In a capital crunch, optimal capital requirements are relaxed to prevent a sharp decline in lending and investment. If productivity decreases, however, they are tightened as preserving financial stability only entails a small cost.

JEL Classification: G21, G28

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I am grateful to Christian Keuschnigg for helpful discussions and comments. Financial support of the Swiss National Science Foundation (Project no. 100018_146685/1) is gratefully acknowledged.

1 Introduction

The interplay of the banking system in general and capital regulation in particular with the business cycle has figured prominently in the context of banking reform and is one of the main aspects in several policy reports.[1] The fundamental problem is well known: During a downturn, many banks experience negative funding shocks as, for example, more frequent loan losses weaken their capitalization while it is particularly challenging to raise new equity such that regulatory constraints become binding. At the same time, traditional, risk-sensitive capital requirements tighten as risk weights increase to account for the generally higher loan risk. In order to meet the regulatory requirements, banks thus deleverage and cut lending, which may even lead to a credit crunch. This clearly procyclical behavior aggravates the downturn with a potentially adverse feedback on financial stability. Yet, bank loans are riskier in bad times such that a larger capital buffer is necessary to prevent a costly banking crisis. In addition, the investment prospects in the real sector are often rather poor, and a smaller loan supply as a result of binding regulatory constraints may thus turn out to be less problematic because fewer investments would be realized even if funding was available. The conflicting goals of ensuring bank safety and preventing a further decline of investment and aggregate demand to some extent reflect the tension between micro- and macroprudential regulation. With Basel III, regulators try to mitigate the procyclicality of capital requirements through a countercyclical and a capital conservation buffer.[2] As a result, regulation tends to be tougher in good times when the risk of unsustainable lending booms and asset price bubbles is high and more relaxed in bad times when recapitalization is difficult.

This paper provides a normative analysis of how capital requirements should adjust to different (macro-)economic shocks. It presents a model of the optimal capital structure where equity provides a buffer against loan losses and thus lowers the risk of bank failure, which entails a social cost. As an innovation, we explicitly model the real sector consisting of bank-dependent entrepreneurs thereby endogenizing the loan market. This approach reveals important equilibrium effects that influence the optimal adjustment and allows identifying real sector determinants of capital regulation. At the core of this paper is an extensive comparative statics analysis with two scenarios: (i) a shortage of bank

[1]For example, Brunnermeier et al. (2009), Turner Review (2009), and FSB (2009).
[2]See sections III and IV in BCBS (2010).

capital (henceforth: capital crunch) that limits banks' lending capacity and (ii) a lower productivity of entrepreneurs that reduces loan demand and the value of investment. The optimal capital requirements, which balance the trade-off between the stability of banks and the ability of entrepreneurs to finance profitable investments, relate to state of the economy through the lending rate, which acts as a *de facto* substitute for equity, and the welfare-maximizing level of bank risk. Their adjustment fundamentally differs between the two scenarios: Capital requirements should be relaxed in a capital crunch to prevent a contraction of lending but they should be stricter if productivity declines such that the lending rate and the value of investment are low. Importantly, optimal regulation allows the economy to adjust at two margins - risk and lending - whereas one of them is fixed under risk-sensitive or flat capital requirements.

The analysis builds on the literature on the real effects of capital regulation and, more generally, of funding shocks[3]: Since the introduction of the Basel accords, their real and especially their procyclical effects have been extensively studied.[4] As a first benchmark, the Modigliani-Miller theorem, however, implies that capital requirements do not have any pronounced real effects as they can be fulfilled with outside equity which should not raise the cost of capital. Such arguments have recently been emphasized, for instance, by Admati et al. (2011); quantitative simulations by Miles et al. (2012) imply only minor long-run effects on customers' borrowing cost even if capital requirements strongly increase. Nevertheless, equity can be scarce and expensive[5] especially during bad times such that capital requirements have the potential to affect lending and investment. Blum and Hellwig (1995) highlight two key frictions that create such real effects: First, banks do not recapitalize by issuing new equity and deleverage instead, second, firms cannot fully substitute bank loans with other funds. They show that whenever capital requirements are binding, equilibrium output and prices become more sensitive to aggregate demand shocks thereby amplifying macroeconomic fluctuations. Furthermore, Heid (2007) shows that banks may hold voluntary buffers in excess of capital charges. These buffers mitigate but do not offset the procyclical effects of capital requirements. Further theoretical

[3]A seminal theoretical contribution is Holmström and Tirole (1997) who study the (heterogeneous) effects of shocks to the supply of different types of capital.

[4]For an overview about links between capital requirements and the real economy, see, Goodhart and Taylor (2006).

[5]For example due to tax benefits of debt finance or asymmetric information cost of equity and signaling considerations as emphasized by Myers and Majluf (1984).

contributions on the procyclicality of capital regulation include, among others, Estrella (2004), Zhu (2008), and Covas and Fujita (2010). On the empirical side, early evidence of how binding regulatory constraints affect lending is provided by Peek and Rosengren (1995a) who study the New England capital crunch in the early 1990s when capital requirements were actively enforced. They find that assets of banks subject to formal enforcement actions shrink significantly faster than those of banks without and that loans to bank-dependent borrowers are most strongly affected. Using a sample of French firms, Fraisse et al. (2013) find that a one percentage point increase in bank capital requirements lowers credit by eight and firm borrowing by four percent. Hence, firms can partly but not fully compensate the smaller loan supply. In a similar spirit, Aiyar et al. (2014) present evidence for the UK and stress the role of loans from foreign banks as substitutes. The procyclicality of capital requirements, in particular of Basel II, and their amplification effects are documented, for example, by Kashyap and Stein (2004) and Gordy and Howells (2006) for American, Repullo et al. (2010) for Spanish, and Andersen (2011) for Norwegian banks.

This paper contributes to the literature on the optimal adjustment of bank regulation to macroeconomic shocks: Kashyap and Stein (2004)[6] show that if the shadow value of bank capital varies over the cycle, optimal capital requirements should be countercyclical. More precisely, they argue for a family of risk curves, which map the risk of each asset into a capital charge, where each curve is associated with a specific shadow value. This preserves the sensitivity of capital requirements across asset categories with different risks but allows for an adjustment over the cycle. In a dynamic equilibrium model with time-varying loan risk, Repullo and Suarez (2013) compare the welfare properties of different regulatory systems and conclude that optimal capital requirements are procyclical but that their variation is less pronounced than that of Basel II for sufficiently large values of the social cost of bank failure. Several contributions analyze capital regulation in models with agency problems: Dewatripont and Tirole (2012) show that capital requirements allocate the control rights of bank shareholders and debtholders as to ensure managerial effort and prevent gambling for resurrection. Regulation should neutralize macroeconomic shocks that would otherwise distort incentives; this is achieved by countercyclical capital buffers or capital insurance. Repullo (2013) stresses the role of costly bank capital

[6]The underlying model can be found in the 2003 working paper version.

in a risk-shifting model: He studies the trade-off between mitigating risk shifting and preserving the lending capacity of banks. Given a shortage of bank capital, its shadow value increases and optimal capital requirements are relaxed. If they remained unchanged, banks would be safer but aggregate investment would sharply drop. Focusing on credit cycles, Gersbach and Rochet (2012) argue that countercyclical capital regulation implemented, for example, as an upper bound on short-term debt corrects the misallocation of credit between good and bad states of nature thereby dampening fluctuations. Several options how cyclically-varying capital regulation can be implemented have been suggested, in particular, direct and indirect smoothing rules for capital requirements [e.g., Gordy and Howells (2006), Brunnermeier et al. (2009), Repullo et al. (2010)] and the build-up of countercyclical buffers [e.g., FSB (2009), BCBS (2010)], which are envisaged by Basel III. Alternative proposals include dynamic provisioning, contingent convertibles and capital insurance [e.g., Kashyap et al. (2008)], and regulatory discretion. Yet, it is too early to present evidence about the consequences of such countercyclical measures but Jiménez et al. (2015) evaluate a comparable policy introduced in Spain already in 2000: dynamic provisions. These provisions are built up from retained earnings during a boom to cover loan losses in bad times where equity is scarce, and they are, in fact, similar to countercyclical capital buffers. They find that dynamic provisions significantly mitigate the fall of bank lending and firm borrowing during the financial crisis. In the good times during the early 2000s, banks that had to build up larger provisions reduced their loan supply but firms could easily substitute by borrowing from less affected banks.

The main contribution of this paper is a comprehensive study of how optimal capital requirements adjust to changes in (macro-)economic conditions especially during a downturn. A full-fledged model of the real sector and the loan market identifies equilibrium effects associated with the lending rate that together with changes in the optimal risk level determine the regulatory adjustment. In addition, this extension allows analyzing the response to productivity shocks, which have not been studied so far despite their importance in macroeconomics. The model is most closely related to Repullo (2013) to which we add two innovations: the model of the real sector and the role of bank capital as a buffer. The latter is more conventional than the incentive effect but requires positively correlated loan returns.

The remainder of this paper is organized as follows: Section 2 outlines the model. Section 3 characterizes the equilibrium and analyzes optimal capital requirements and its adjustment. It also provides a numerical example. Finally, section 4 concludes.

2 The Model

We develop a static, partial equilibrium model of the optimal capital structure of banks. The economy is populated by four types of risk-neutral agents: entrepreneurs representing the real sector and banks, investors (bank shareholders), and depositors representing the financial sector. Banks attract deposits and equity from depositors and investors and provide loans to entrepreneurs, who can invest in profitable but risky projects. Whenever the project fails, the entrepreneur defaults and the bank incurs a loan loss. The risk characteristics crucially depend on whether the economy experiences a recession, which is revealed after projects were initiated: Usually, only idiosyncratic risk matters such that the bank can diversify its loan portfolio. In a recession, however, systemic risk materializes and the defaults of entrepreneurs are positively correlated. As a result, a bank may fail whenever too many borrowers simultaneously default and its equity cannot fully absorb all losses. Bank failure entails social costs that are not internalized by banks and thus provide a rationale for regulation. The timing is as follows: (i) banks attract capital from depositor and investors and provides loans to entrepreneurs who invest, (ii) it is revealed whether project risks are independent (normal state) or positively correlated (recession), and (iii) the projects mature and the contracts are settled.
The following friction ensures that capital requirements have the potential to affect the real economy:

ASSUMPTION 1 *Entrepreneurs can finance their projects with bank loans only.*

Hence, entrepreneurs are bank-dependent and do not raise funds directly from investors or depositors. Evidence of Fraisse et al. (2013) and Jiménez et al. (2015) supports this assumption especially during bad times. In a broader context, this of course concerns only some firms like, for example, small businesses, while others can access the capital market. Another friction - whether banks raise new equity or deleverage to satisfy capital requirements - endogenously emerges depending on the scarcity of bank capital.

2.1 Entrepreneurs

The real sector consists of a continuum of measure one of penniless entrepreneurs. Each of them can undertake a risky investment project characterized by:

ASSUMPTION 2 *The unit-size project yields a binary return*

$$\widetilde{R} = \begin{cases} R, & 1 - p_0 \\ \alpha, & p_0 \end{cases}$$

with $R > 1 > \alpha$. *The net present value is positive:* $\mu \equiv (1 - p_0)R + p_0\alpha - 1 > 0$.

Subsequently, we interpret the return R as the entrepreneur's productivity and α as the liquidation value. If the project fails, the latter is appropriated by the lender and $1 - \alpha$ equals the loss given default. Failure and success probability, p_0 and $1 - p_0$, are *ex ante* probabilities that consist of an idiosyncratic and a systemic component; the latter allows for correlated failures.

The loan demand is modeled as in Repullo and Martinez-Miera (2010): Entrepreneurs face heterogeneous opportunity cost, $u \sim U[0, 1]$. They may, for instance, represent forgone labor income or the value of leisure. Only entrepreneurs whose opportunity cost are smaller than the expected net return on investment borrow and invest:

$$u \leq (1 - p)(R - r_L) \equiv \hat{u}(r_L) \tag{1}$$

$\hat{u}$ defines the marginal entrepreneur who is just indifferent between investing and choosing the outside option. Since opportunity cost are uniformly distributed, $\hat{u}$ also equals the fraction of investing entrepreneurs (i.e., with opportunity cost below the threshold) and thus the loan demand, which decreases in the lending rate r_L and the *ex ante* project risk p_0 and increases in productivity R. The surplus of an active entrepreneur equals $(1-p_0)(R-r_L)-u$; the aggregate surplus of the real sector is $\pi^E = \int_0^{\hat{u}}(1-p_0)(R-r_L)-udu$. Since they undertake a single investment, the correlation of projects matters little for individual entrepreneurs. However, it is instrumental for banks as they may fail whenever too many entrepreneurs simultaneously default. We suggest an intuitive and tractable model of project correlation across entrepreneurs: The economy may experience either normal conditions or a recession, which is revealed after the projects are initiated and determines to what extent failures are independent or correlated:

ASSUMPTION 3 *In normal times projects (probability $1-\theta$) are independent and each of them succeeds with probability $1-p$ and fails with probability p. In a recession (probability θ) a fraction x of projects immediately fails where $x \in [0,1]$ is distributed according to some continuous, differentiable distribution function $F(x)$. The remaining projects continue and succeed with probability $1-p$ and fail with probability p*

Hence, x captures the systemic and p the idiosyncratic component. Projects are generally independent but a recession is associated with an adverse shock to a stochastic number of projects, which thus immediately fails. This represents a macroeconomic shock that has the potential to affect all entrepreneurs at the same time like, for example, a fall in aggregate demand or - in a small, open economy - a sudden appreciation of the currency. Figure 1 illustrates the possible outcomes: In normal times, the project succeeds with probability $1-p$ and fails with probability p. In a recession, a fraction x of all projects fails due to the shock, whereas the projects unaffected by the shock either succeed (share $1-p$) or fail (share p) according to idiosyncratic risk. The stochastic variable x measures the severity of a recession, high realizations point to a severe recession.

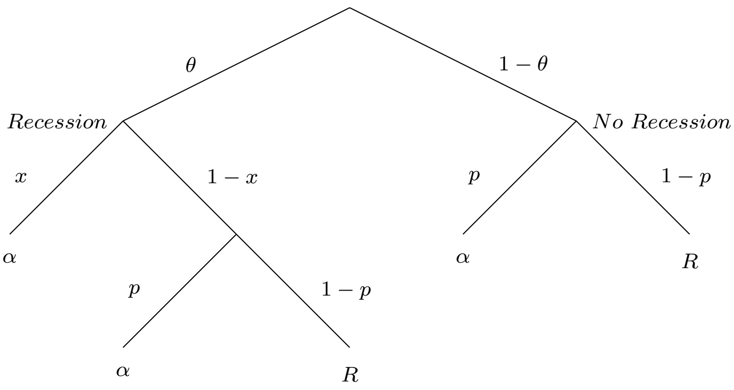

Figure 1: *Probability Tree*

Eventually, table 1 summarize a project's success and failure probabilities *ex ante* as well as in a recession and in normal times using $x_0 \equiv E(x) = \int_0^1 x dF(x)$. Intuitively, a recession revises the failure probability up compared to the project-specific failure probability p.

	Success	Failure
Ex ante	$1 - p_0 = (1-p)(1-\theta x_0)$	$p_0 = p + \theta x_0(1-p)$
Recession	$(1-p)(1-x)$	$p + x(1-p)$
No Recession	$1-p$	p

Table 1: *Probabilities*

2.2 Banks

There is a continuum of measure one of banks that lend to entrepreneurs; more specifically, they provide unit-size loans to a mass L of active entrepreneurs. Each bank can raise funds from two sources: deposits (share $1-k$) and bank capital (share k). Bank owners are protected by limited liability. Deposits are elastically supplied at the risk-free (gross) interest rate normalized to one but depositors require a compensation for bearing the bank's failure risk giving rise to a risk-adjusted deposit rate $r \geq 1$. One might alternatively interpret r as the risk-free rate plus an actuarially fair deposit insurance premium. Bank capital is provided by investors (i.e., outside shareholders) who require an expected (gross) return on equity $\gamma \geq 1$.

Bank risk crucially depends on whether the economy is in a recession or not: In general, loans are uncorrelated because a fraction $1-p$ is repaid and a fraction p fails. Hence, the portfolio is diversified and the bank is safe. In a recession, however, a stochastic fraction $p+x(1-p)$ of loans fail, namely, a share x due to the adverse shock and a share $(1-x)p$ due to project-specific risk. The bank thus receives the full repayment r_L from a fraction $(1-x)(1-p)$ of borrowers and the liquidation value α from a fraction $p+(1-x)p$. It succeeds as long as enough loans are repaid which requires the share of entrepreneurs who receive an adverse shock to be smaller than the failure threshold $\hat{x}$ given by:

$$(1-\hat{x})(1-p)r_L + [p + \hat{x}(1-p)]\alpha - r(1-k) = 0 \tag{2}$$

Hence, the liabilities of the bank, $r(1-k)L$, are just covered by the assets consisting of repaid and liquidated loans. In other words, the bank's end-of-period equity is zero. Obviously, the failure threshold increases in the deposit rate and in idiosyncratic project risk and decreases in the capital ratio, the lending rate, and the liquidation value. Importantly, the deposit rate is endogenous because depositors require a risk-adjusted interest rate. As soon as the recession is more severe and a larger number of borrowers defaults, the loss is so large that the bank's equity is negative and its liabilities are not fully covered. Bank failures are correlated because banks are identical and defaults correlated, which gives rise to a systemic banking crisis if $x > \hat{x}$. Consequently, the banks survive in a mild recession when only a few entrepreneurs default due to the shock. Whenever the shock is more severe, banks fail such that the recession aggravates to a systemic banking crisis. The *ex ante* probabilities of a mild recession and a banking crisis are $\theta F(\hat{x})$ and $\theta[1 - F(\hat{x})]$ respectively. The latter also corresponds to the probability of bank failure. Since bank owners are protected by limited liability such that their payoff is zero in case of failure, the bank's expected surplus is:

$$\begin{aligned}\pi^B = \theta \int_0^{\hat{x}} (1-x)(1-p)r_L + [p + x(1-p)]\alpha - r(1-k)dF(x)L \\ +(1-\theta)[(1-p)r_L + p\alpha - r(1-k)]L - \gamma kL\end{aligned} \tag{3}$$

It consists of the expected profit in a recession (with probability θ) and in normal times (with probability $1-\theta$) net of the required return on equity. In both states, the profit equals gross interest income from repaid and the liquidation value of failed loans minus deposit repayment. To maximize its surplus, the bank determines the capital structure (i.e., the capital ratio k) and the loan supply L.

Eventually, we add the assumption that bank failure is costly for society and rely on reduced-form social cost characterized by:

ASSUMPTION 4 *A banking crisis entails a social cost c per unit of loans.*

These costs represent, for example, the cost of bank runs, the loss of lender-borrower relationships or disruptions to the payment systems.[7] The failure of banks to internalize these cost is the reason why the market equilibrium is inefficient, which provides a rationale for capital regulation. This is a common motivation in the literature applied, for instance, by Kashyap and Stein (2004), Repullo (2013), and Repullo and Suarez (2013).

2.3 Depositors and Investors

The supply side is modeled as in Repullo (2013) with an elastic deposit and a fixed bank capital supply: On the one hand, risk-neutral depositors elastically supply deposits as

[7]Note that wealth losses of depositors (or the cost of providing deposit insurance) are fully internalized as deposits are correctly priced.

long as they yield an expected return equal to the (gross) risk-free interest rate that is normalized to one. Hence, there is market discipline as the interest rate compensates depositors for bearing the bank's failure risk[8] such that:

$$E\left[\min\left\{r, \frac{(1-p)(1-x)r_L + [p + x(1-p)]\alpha}{1-k}\right\}\right] = 1 \tag{4}$$

One may interpret this condition as the participation constraint of depositors: Whenever bank succeeds, the bank pays an interest rate r. In case of failure, however, each depositor inherits a share $\frac{1}{(1-k)L}$ of its assets $[(1-p)(1-x)r_L + (x + (1-x)p)\alpha]L$. Consequently, depositors earn the deposit interest rate in in normal times or in a mild recession (which occur with probability $1-\theta$ and $\theta F(\hat{x})$ respectively) and inherit the bank's assets in a banking crisis:

$$[1-\theta+\theta F(\hat{x})]r + \theta \int_{\hat{x}}^{1} \frac{(1-p)(1-x)r_L + [p + x(1-p)]}{1-k} dF(x) = 1 \tag{5}$$

Since they are paid a risk-adjusted interest rate, depositors' expected surplus is zero: $\pi^D = 0$. As long as the deposit rate satisfies the participation constraint, they are willing to supply any quantity.

On the other hand, investors supply an amount K of bank capital and require an expected return on equity γ which is at least one:

$$K(\gamma) = \begin{cases} K, & \text{if } \gamma \geq 1 \\ 0, & \text{if } \gamma < 1 \end{cases}$$

Hence, the expected surplus of investors is $\pi^I = (\gamma - 1)K(\gamma) \geq 0$. Whenever the supply is small, bank capital is scarce such that a trade-off emerges between financial stability in the sense of a low bank risk and lending and investment. A fixed supply of bank capital is typical for models of funding shocks and the effects of capital regulation such as Holmström and Tirole (1997) and Repullo (2013). This formulation allows capturing such shocks by comparative statics. An alternative is an exogenous excess return on equity such as in Repullo and Suarez (2013).

[8] Alternatively, suppose that deposits are insured and banks pay an actuarially fair insurance premium.

2.4 Markets

In this economy, three markets exist - a market for loans, deposits, and bank capital. The loan market clears as soon as $L = \hat{u}$ such that the loan supply equals the fraction of entrepreneurs who invest. This pins down the lending rate r_L. Given the perfectly elastic supply, the deposit market is in equilibrium whenever banks promise a deposit rate that satisfies the participation constraint of depositors (4). Eventually, the market for bank capital is in equilibrium if $K(\gamma) = kL$ thereby determining the return on equity. However, this market may not clear if bank capital is abundant in supply such that $K > kL > 0$ even if the required returns on equity and deposits are the same ($\gamma = 1$).

2.5 State of the Economy

The state of the economy characterizes the (macro-)economic conditions. We examine the optimal adjustment of capital requirements to a financial and a real shock and focus on two parameters: the availability of bank capital[9] given by the fixed supply K and entrepreneurs' productivity R. The supply of bank capital, first of all, affects banks. As soon as they face binding capital requirements and borrowers are bank-dependent, a shortage of bank capital - a capital crunch - may force banks to cut lending and deleverage, which has real effects as it limits entrepreneurs' investment. he empirical relevance of capital crunches is documented, for example, by Bernanke and Lown (1991) and Peek and Rosengren (1995b). Such a scenario is also at the core of Repullo's (2013) analysis. This scenario represents a financial shock that can be the result of swings in investors' moods, optimism and risk aversion.

The project return R, in contrast, characterizes entrepreneurs' investment prospects and captures technology or productivity shocks that feature prominently in macroeconomics. It is a crucial determinant of entrepreneurs' investment decisions and thus influences the loan demand. Optimal regulation may adjust because of equilibrium effects associated with the lending rate and changes in the value of projects that influence the underlying trade-off between financial stability and investment.

[9]In our static setting where banks raise new equity, the interpretation of changes in the supply of bank capital appears suitable. In a dynamic model, a broader interpretation would also include shocks to the current capitalization, for example, due to loan losses.

3 Equilibrium Analysis

This section characterizes two allocations: the market equilibrium and the social optimum where all costs associated with bank failure are internalized. The latter is the reason for market failure in the sense that banks are inadequately capitalized and may provide too large an amount of loans. Subsequently, we show how the optimal allocation can be decentralized using capital requirements and study their adjustment to economic shocks. An outcome of key interest in both allocations is bank risk: It is jointly determined by the identity that equalizes assets and liabilities (2) and the participation constraint of depositors (4) that pin down the failure threshold and the deposit rate respectively:

LEMMA 1 *A bank fails in a recession if a fraction $x > \hat{x}$ of entrepreneurs immediately default. This threshold is characterized by*

$$1 - k - \alpha - (r_L - \alpha)(1-p)H(\hat{x}) = 0 \tag{6}$$

where

$$H(\hat{x}) = (1-\theta)(1-\hat{x}) + \theta \int_{\hat{x}}^{1} F(x)dx$$

is a decreasing function of $\hat{x}$ with $H'(\hat{x}) = -[1 - \theta + \theta F(\hat{x})] < 0$, $H(0) = 1 - \theta x_0$ and $H(1) = 0$. The failure threshold increases in the capital ratio, the lending rate, and the liquidation value.

Proof: See Appendix A.1.

Condition (6) relates bank risk to the capital structure and the lending rate. Well-capitalized banks that earn a high lending rate are particularly safe. Moreover, a bank can be risk-free whenever it succeeds in repaying deposits even if all borrowers simultaneously default (i.e., the default rate equals $\hat{x} = 1$). This requires a capital ratio of (at least) $1 - \alpha$, which suffices to cover the loss given default. In the extreme case $\alpha = 0$, this would require an all-equity financed bank.

3.1 Market Equilibrium

The market equilibrium provides a benchmark: Each bank determines its capital structure k and loan supply L as well as the interest rate offered to depositors r in order to maximize the expected surplus π^B which is given in (2) subject to depositors' participation constraint (5). By substituting the latter into the objective function to eliminate r, one obtains the consolidated problem:

$$\pi^B = \max_{k,L} \left[(1-p_0)r_L + p_0\alpha - (1-k) - \gamma k\right] L \tag{7}$$

The bank's optimal choices are summarized in

LEMMA 2 *The bank's capital ratio is indeterminate, $k \in [0,1]$, if $\gamma = 1$ and zero, $k = 0$, if $\gamma > 0$. The loan supply is elastic at the lending rate*

$$r_L = \frac{1-p_0\alpha}{1-p_0} \tag{8}$$

such that banks earn a zero expected surplus: $\pi^B = [(1-p_0)r_L + p_0\alpha - 1]L = 0$.

Proof: Substituting the participation constraint of depositors (5) into the objective function of the bank (2) yields the consolidated problem (10). The indeterminacy of the capital structure follows from the first-order condition $\frac{\partial \pi^B}{\partial k} = 1 - \gamma \leq 0$; $\frac{\partial \pi^B}{\partial L} = (1-p_0)r_L + p_0\alpha - (1-k) - \gamma k = 0$ implies that banks provide loans until they earn a zero expected surplus; substituting either $\gamma = 1$ or $k = 0$ gives (8). *Q.E.D.*

The loan supply is elastic because of the linear technology and the elastic supply of deposits. Hence, the lending rate exactly compensates banks for bearing the project risk leading to zero expected profits. In other words, the bank itself (i.e., the inside shareholders) does not earn a rent. If it incurred convex cost or relied on scarce equity, however, the loan supply would not be perfectly elastic. In line with Modigliani-Miller, the capital structure is indeterminate: Since equity has no advantage over debt because the latter is correctly priced such that wealth losses of depositors are fully internalized, the bank is indifferent as long as both types of capital have the same cost. Whenever bank capital is more expensive (i.e., $\gamma > 1$), the capital ratio is zero. Hence, only a required return on equity of one is consistent with equilibrium such that at least some banks have a positive capital ratio. The irrelevance of the capital structure is of course a strong result: It would disappear in the presence of government guarantees or tax distortions, which imply a strict preference for debt, or bank borrowing frictions (e.g., limited pledgeable income).

Entrepreneurs decide about investment: A project is undertaken if the expected profit exceeds the idiosyncratic opportunity cost u. Derived from the extensive margin, loan demand equals the fraction of entrepreneurs with sufficiently low opportunity cost, $\hat{u}(r_L)$, defined in (1). Together with loan market equilibrium, $L = \hat{u}(r_L)$, this yields:

LEMMA 3 *Equilibrium lending and investment is:*

$$L = (1 - p_0)R + p_0\alpha - 1 = \mu \tag{9}$$

It increases in the project return and is insensitive to the bank capital supply.

Proof: Investment immediately follows from the market clearing condition, $L = \hat{u}(r_L)$, by substituting (1) and (8) for loan demand and lending rate. *Q.E.D.*

Investment equals the project's expected net return: This follows from entrepreneurs' investment choice at the extensive margin combined with the risk-adjusted lending rate (8). The latter guarantees that entrepreneurs earn the project's expected net return, $(1 - p_0)(R - r_L) = \mu$, such that a fraction $\hat{u} = \mu$ of them invests.

3.2 The Regulator's Problem

In the market equilibrium, the social cost of a banking crisis, $C = \theta[1 - F(\hat{x})]cL$, is not internalized. Welfare W consists of the expected surplus of all agents net of the social cost, $W = \pi^E + \pi^B + \pi^I - C$. Substituting $K(\gamma) = kL$, eliminating the deposit rate in (5) and (6), and using $E(x) = p$ yields:

$$W = \int_0^{\hat{u}} (1 - p_0)(R - r_L) - u du + \left[(1 - p_0)r_L + p_0\alpha - 1 - \theta(1 - F(\hat{x}))c\right] L \tag{10}$$

The first term is expected surplus of the real sector; the second term captures the surplus of the financial sector (i.e., of banks and investors) net of the social cost associated with bank failure. The regulator determines lending and investment, the marginal entrepreneur, the capital ratio of banks, and the lending rate in order to maximize welfare thereby fully internalizing the social cost. Recall that the lending rate does affect welfare because of its effect on bank risk, which matters whenever failure entails a cost. In principle, the lending rate should thus be as high as possible to minimize bank risk but it is restricted: The marginal entrepreneur, $\hat{u}$, needs to earn a zero surplus in order to invest. This adds a participation constraint of entrepreneurs. Substituting $L = \hat{u}$, which holds in equilibrium, the optimization problem of a welfare-maximizing regulator is:

PROGRAM 1 *The regulator determines lending and investment L, banks' capital ratio k, and the lending rate r_L to maximize social welfare*

$$\max_{k,L,r_L} [\mu - \theta(1 - F(\hat{x}))c]L - \frac{L^2}{2} \tag{11}$$

subject to the participation constraint of entrepreneurs, $(1 - p_0)(R - r_L) = L$, and the capital availability constraint, $K \geq kL$.

In contrast to Modigliani-Miller's irrelevance theorem, the capital structure has welfare consequences as a higher capital ratio reduces bank risk and thus the social cost of failure. The capital ratio is chosen according to the first-order condition

$$c\theta f(\hat{x}) \underbrace{\frac{1}{(r_L - \alpha)F(\hat{x})}}_{\frac{\partial \hat{x}}{\partial k}} = \lambda_1 \tag{12}$$

where λ_1 is the Lagrange multiplier of the capital availability constraint. The left-hand side captures the marginal gains of a higher capital ratio, namely, the lower risk and failure cost. The marginal cost equal the shadow value of bank capital captured by the multiplier. By the Envelope theorem, the latter measures the welfare contribution of bank capital $\frac{\partial W}{\partial K} = \lambda_1$. Whenever equity at least covers the loss given default, $k \geq 1 - \alpha$ (see lemma 1), and banks are safe, we have $\frac{\partial \hat{x}}{\partial k} = 0$ such that the shadow value of bank capital is zero and additional equity has no welfare effect.

Lending and investment are determined according to the first-order condition

$$\mu - L - \theta[1 - F(\hat{x})]c - \lambda_1 k - \lambda_2 = 0 \tag{13}$$

where λ_2 denotes the Lagrange multiplier of the participation constraint. Intuitively, the marginal welfare gains from lending (i.e., the expected return of financing an additional project) equal the marginal cost consisting of the opportunity and social failure cost. Expansion also tightens both the participation and the capital availability constraint thereby raising bank risk due to a reduction of lending rate or capital ratio.

3.3 Equilibrium Allocation

Based on the first-order conditions and constraints of program 1, one can derive the socially optimal allocation:

PROPOSITION 1 *The failure threshold* $\hat{x}^*$ *and bank lending* L^* *are jointly determined by the system:*

$$J^1(L^*, \hat{x}^*) = \mu - L^* - \theta[1 - F(\hat{x}^*)]c - \frac{c\theta f(\hat{x}^*)}{1 - \theta + \theta F(\hat{x}^*)} \left[\frac{1 - \alpha + \frac{H(\hat{x}^*)L^*}{1-\theta x_0}}{\left(R - \frac{L^*}{1-p_0} - \alpha\right)(1-p)} - H(\hat{x}^*) \right] = 0 \quad (14)$$

$$J^2(L^*, \hat{x}^*) = K - \left[1 - \alpha - \left(R - \frac{L^*}{1 - p_0} - \alpha\right)(1-p)H(\hat{x}^*)\right] L^* = 0 \quad (15)$$

Lending $L^* \leq \mu$ *increases in the supply of bank capital,* $\frac{\partial L^*}{\partial K} \geq 0$*, and productivity,* $\frac{\partial L^*}{\partial R} > 0$*. The failure threshold* $\hat{x}^*$ *increases in the supply of bank capital,* $\frac{\partial x^*}{\partial K} \geq 0$*, but may increase or decrease in productivity. This allocation requires the capital ratio:*

$$k^* = 1 - \alpha - \left(R - \frac{L^*}{1 - p_0} - \alpha\right)(1-p)H(\hat{x}^*) \quad (16)$$

Proof: See Appendix A.1.

Compared to the market equilibrium, condition (14) implies that lending and investment are usually smaller and the lending rate is higher. The reason is that internalizing the social cost of a banking crisis requires the use of scarce bank capital.

Essentially, the optimal allocation trades off the benefit of a lower bank risk against smaller lending and investment. This trade-off emerges as long as bank capital, which is necessary to improve the stability and resilience of banks, is scarce. Therefore, a larger supply relaxes the capital availability constraint such that more investments is financed without increasing risk and bank risk decreases by improving their capitalization. An increase in entrepreneurs' productivity, in contrast, makes the projects more valuable thereby tilting the trade-off more in favor of investment. In order to mobilize additional funds, the failure threshold of banks likely falls such that the failure risk is higher. This outcome materializes as long as capital requirements are tight and the risk of failure is low. The optimal capital structure given by (16) ensures that the balance sheet of each bank is consistent with the socially optimal failure threshold $\hat{x}^*$. This is the reason why the capital structure is not irrelevant à la Modigliani-Miller. Instead, the capital ratio mechanically follows from the definition of the failure threshold, (6), and implements the optimal risk level.

The supply of bank capital can be large enough to make banks risk-free such that they succeed in a recession so severe that all loans fail ($x = 1$) and only the liquidation value is recovered:

COROLLARY 1 *If the supply of bank capital exceeds* $K \geq (1-\alpha)\mu \equiv K_0$*, the capital ratio is* $k^* = 1 - \alpha$ *such that banks are risk-free,* $\hat{x}^* = 1$*, and lending and investment are similar to the market equilibrium,* $L^* = \mu$*.*

Proof: A risk-free bank (i.e., $\hat{x} = 1$) requires $k \geq 1 - \alpha$ (see lemma 1); it implies $\frac{\partial \hat{x}}{\partial k} = \frac{\partial \hat{x}}{\partial r_L} = 0$ such that $\lambda_1 = \lambda_2 = 0$. Hence, $L = \mu$ follows from (14) and the participation constraint of depositors requires $r_L = (1 - p_0\alpha)/(1 - p_0)$. This outcome is only feasible if $K \geq K_0$ according to the capital availability constraint (15). *Q.E.D.*

Although lending and investment are similar to the unregulated market equilibrium, the latter is not necessarily efficient because the capital structure is indeterminate: On average, banks' capitalization may be sufficient but in the absence of regulation it is possible that some banks have too small a capital ratio $k < 1 - \alpha$ and are risky. Whenever bank capital is abundant in supply ($K \geq K_0$), the trade-off between financial stability and real investment disappears and a high capital ratio does not entail any costs for the real economy. This case is consistent with the key argument of Admati et al. (2011).

3.4 Optimal Capital Regulation

Internalizing the social cost of a systemic banking crisis provides a rationale for regulation, which allows decentralizing the optimal allocation in a market economy. Essentially, the regulator requires that banks have the optimal capital structure:

COROLLARY 2 *The optimal allocation can be implemented by minimum capital requirements*

$$k \geq 1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}^*) \equiv k^*(r_L, x^*) \tag{17}$$

that increase in the failure threshold and decrease in the lending rate and the liquidation value. Capital requirements bind if $K \leq K_0$*.*

Proof: See Appendix A.1.

The capital requirements are a function of the optimal failure threshold $\hat{x}^*$ (henceforth: target risk), the lending rate r_L and the liquidation value α: A lower target risk naturally requires more equity, while a higher lending rate and liquidation value allow reducing the capital ratio without leading to higher bank risk. They increase the bank's income, $(1-x)(1-p)r_L + (p + (1-x)p)\alpha$, thereby providing an additional buffer such that the same failure risk materializes even with a smaller capital ratio. Consequently, a high lending rate or liquidation value are substitutes for capital on the 'risk front'. A similar substitution effect is found by Repullo and Suarez (2013) for the capital structure of banks that hold voluntary buffers because of potentially binding regulatory constraints in the future. Therefore, the state of the economy - entrepreneurs' productivity and the supply of bank capital - influences capital requirements through two channels: (i) target risk and (ii) equilibrium lending rate. While the former is optimally chosen by the regulator, the latter is determined by the market. As soon as the supply of bank capital is large enough to make banks risk-free without limiting investment (i.e., $K \geq K_0$), however, capital requirements equal the loss given default $k = 1 - \alpha$ and are insensitive to economic shocks.

Implementing the optimal allocation (proposition 1) with capital requirements is straightforward: The optimal capital structure varies with the lending rate and thus ensures by construction that a bank's failure threshold is indeed $\hat{x}^*$. In addition, it implements the optimal lending scale L^*: Banks maximize their expected surplus π^B subject to the regulatory constraint $k \geq k^*(r_L, \hat{x}^*)$. From the first-order condition with respect to loans $(1-p_0)r_L + p_0\alpha - 1 - (\gamma-1)k = 0$, the loan demand of entrepreneurs $\hat{u} = (1-p_0)(R-r_L)$, and loan market clearing $L = \hat{u}$, one finds that loans

$$L = \mu - (\gamma - 1)k \tag{18}$$

decrease in the capital ratio and in the required return on equity. If equity earns no excess return over debt (i.e., if $\gamma = 1$), however, lending is independent of the capital structure and similar to the market equilibrium. Together with market clearing for bank capital, $K = kL$, this condition determines equilibrium lending and return on equity: Since capital requirements are binding, market clearing coincides with the capital availability constraint in the regulator's program. Therefore, the lending is scale optimal: $L = L^*$.[10] The return on equity endogenously adjusts: As long as bank capital is scarce, $K < K_0$, such that $k^* < 1 - \alpha$, the market for bank capital clears thereby determining loans $L = L^* < \mu$. From condition (18), equity earns an excess return compared to deposits $\gamma > 1$. If $K > K_0$, capital requirements make banks risk-free such that the externality vanishes and maximum lending is optimal $L^* = \mu$. Accordingly, equity earns the same return as deposits, $\gamma = 1$, and the lending rate equals $r_L = \frac{1-p\alpha}{1-p}$ like in the market equilibrium.[11]

3.4.1 Capital Requirements and the State of the Economy

This section studies the optimal adjustment of capital requirements in two different scenarios: a capital crunch, that is, a contraction of the bank capital supply K, and an adverse shock to entrepreneurs' productivity R. The capital crunch captures a key concern in the procyclicality debate[12], while productivity shocks play a central role in macroeconomics as one of the driving forces of the business cycle. As discussed above, two channels - target risk and the equilibrium lending rate - link capital regulation to the state of the economy. The adjustment is characterized by:

PROPOSITION 2 *Optimal capital requirements $k^*(r_L, \hat{x}^*)$ increase in the supply of bank capital, $\frac{\partial k(r_L,\hat{x}^*)}{\partial K} > 0$, and decrease in entrepreneurs' productivity, $\frac{\partial k(r_L,\hat{x}^*)}{\partial R} < 0$. Whenever the supply of bank capital is abundant, $K \geq K_0$, capital requirements are independent of the state of the economy.*

Proof: See Appendix A.1.

A larger supply of bank capital allows (i) increasing the loan supply and (ii) raising the capital ratio to reduce bank risk (i.e., raise the failure threshold $\hat{x}$). Proposition 1 shows that a combination of both is optimal, which drives the response of the capital structure: First, a larger supply of loans reduces the equilibrium lending rate given that the demand is unaffected. This lowers the bank's interest revenue such that it can absorb

[10]Since the demand monotonically increases in L and the supply is fixed, the solution is $L = L^*$.

[11]Note that the capital requirements are binding if $\gamma > 1$ but can be slack if $\gamma = 1$; in the second case, banks may choose a higher capital ratio than $1 - \alpha$ but they never lend more than μ.

[12]See, for instance, Repullo (2013).

fewer loan losses. Only a higher capital ratio can preserve the risk level. Second, reducing target risk is optimal, which requires an even higher capital ratio. Both effects clearly imply tighter capital requirements. Conversely, the optimal response to a capital crunch is to relax them: On the one hand, the contraction of the loan supply generates a positive equilibrium effect through a higher lending rate, which allows reducing the capital ratio without affecting bank risk. On the other hand, tolerating a higher risk level is optimal as the decline of investment would otherwise be too strong. Therefore, regulation is countercyclical in the sense that capital requirements are tightened (relaxed) in case more (less) bank capital is available. This countercyclical adjustment is qualitatively similar to Repullo (2013).

A positive technology shock or, more generally, attractive investment prospects increase the value of the projects such that even entrepreneurs with high opportunity cost find it profitable to invest and loan demand increases. Productivity affects capital requirements in two different ways: First, the higher loan demand increases the lending rate and the bank's interest income. This allows for a lower capital ratio without undermining financial stability. Second, tolerating a higher failure risk is usually optimal especially if capital requirements are tight. This implies a further decrease in the capital ratio. Even in case target risk is lower, the effect of a higher lending rate prevails, and the capital ratio unambiguously decreases if entrepreneurs become more productive. The somewhat ambiguous response of bank risk is precisely due to these two counteracting effects: the higher lending rate versus the lower capital ratio. The main purpose of relaxing capital requirements whenever investment opportunities improve is to accommodate the higher loan demand and to ensure that banks can fund more projects despite a fixed capital supply. If investment prospects worsen, in contrast, capital requirements should be tightened to account for the declining lending rate and to exploit the low project value and loan demand in order to maintain or even reduce bank risk. In other words, preserving or even improving financial stability only entails a relatively small cost. Hence, regulation is procyclical in the sense that capital requirements are relaxed (tightened) in case of a positive (negative) productivity shock.

The analysis offers three main insights: First, optimal regulation is related to the state of the economy through target risk and the equilibrium lending rate. Second, the cyclical adjustment fundamentally differs depending on the type of economic shock: Regulation is clearly procyclical in case of productivity shocks but countercyclical with regard to fluctuations in the supply of bank capital. In part, this difference arises because of an opposite equilibrium effect: A contraction of the bank capital supply leads to a higher lending rate as banks reduce their loan supply. This, in turn, allows for a lower capital ratio without raising bank risk. An adverse productivity shock, in contrast, lowers loan demand such that the lending rate falls, which mechanically requires a higher capital ratio to avoid higher bank risk. In particular, the regulator may adjust target risk: In case of a shortage of bank capital, a higher risk should be tolerated, whereas the response to a declining productivity is often to reduce risk. Intuitively, the latter can be achieved at lower cost because of the rather unattractive investment prospects. Third, whenever a downturn involves both declining productivity and a shortage of bank capital at the same time, the optimal adjustment of capital requirements is ambiguous and depends on the relative magnitude of the two effects.

3.4.2 Comparison

This section compares the optimal adjustment to that of two alternative systems: flat and risk-sensitive capital requirements. Defining a constant ratio of bank capital to total assets, the former are similar in kind to the leverage ratio envisaged in Basel III. The latter define a minimum ratio of capital to risk-weighted assets and essentially target a particular risk level such as the target one-year solvency probability of 99.9% in Basel II (corresponding to a failure probability of 0.1% in our framework); risk-sensitive capital requirements remain an essential part of Basel III.

In case banks are subject to flat capital requirements, $k = \bar{k}$, their failure threshold is determined by $1 - \bar{k} - \alpha - (r_L - \alpha)(1-p)H(\hat{x}) = 0$. Thus, bank risk decreases in the capital requirement, the lending rate, and the liquidation value. Taking into account loan demand and market clearing, lending and the return on equity are jointly determined by the bank's first-order condition and the equilibrium in the bank capital market

$$L = \mu - (\gamma - 1)\bar{k}, \quad K = \bar{k}L$$

In particular, lending and investment are simply a multiple of the bank capital supply: $L = \frac{K}{\bar{k}}$. Whenever the supply of bank capital increases, banks increase loans by a factor

$\frac{1}{\bar{k}}$. In contrast, a higher loan demand due to more productive entrepreneurs only increases the lending rate and the return on equity thereby offsetting any quantity response. As soon as the bank capital supply satisfies $K \geq \bar{k}\mu$, the bank chooses $L = \mu$ such that equity earns no excess return, $\gamma = 1$.

In the risk-sensitive system, the failure threshold essentially becomes a policy parameter, $\hat{x} = \hat{x}'$. Intuitively, the regulator sets capital requirements as to achieve a particular probability of a banking crisis. Condition (6) implies:

$$k(r_L, \hat{x}') = 1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}')$$

The only difference to optimal capital requirements is the exogenous target risk.[13] Again, lending and interest rates follow from the bank's first-order condition combined with loan market clearing

$$L = \mu - (\gamma - 1)\left[1 - \alpha - (r_L - \alpha)(1 - p)H(\hat{x}')\right] = 0,$$
$$K = \left[1 - \alpha - (1 - p)(r_L - \alpha) H(\hat{x}')\right] L$$

with $r_L = R - \frac{L}{1-p_0}$. Differentiating market clearing using r_L from the second condition implies that lending increases in both the bank capital supply and entrepreneurs' productivity. The adjustment of capital requirements is driven by changes in the lending rate: Since the latter falls if banks can access more capital such that the loan supply increases, capital requirements are tightened to avoid higher bank risk. In case of a higher productivity, in contrast, the loan demand and lending rate increase and the higher revenue from repaid loans provides an additional buffer implying a lower capital ratio. The cyclical adjustment is thus qualitatively similar to that of optimal capital requirements but it is entirely driven by changes in the equilibrium lending rate, whereas a welfare-maximizing regulator also adjusts the target risk. Hence, one may expect that the adjustment is less pronounced. Again, bank lending equals μ as soon as $K \geq (1 - \alpha)\left(1 - \frac{H(\hat{x}')}{1-\theta x_0}\right)\mu$ such that $\gamma = 1$. In such a case, capital requirements are $k \geq (1 - \alpha)\left(1 - \frac{H(\hat{x}')}{1-\theta x_0}\right)$ and thus independent of the state of the economy.

Table 2 summarizes how the economy adjusts to (i) a capital crunch and (ii) a decline in productivity depending on the regulatory system. Both optimal and risk-sensitive capital

[13]In case the latter was appropriately chosen and adjusted (i.e., if $\hat{x}' = \hat{x}^*$), the allocation would coincide with the optimal allocation.

	Capital Requirements		
	Optimal	Flat	Risk-Sensitive
	Capital Crunch (K $\downarrow$)		
Capital Ratio	$\downarrow$	$\leftrightarrow$	$\downarrow$
Failure Risk	$\uparrow$	$\downarrow$	$\leftrightarrow$
Lending	$\downarrow$	$\downarrow$	$\downarrow$
	Adverse Productivity Shock (R $\downarrow$)		
Capital Ratio	$\uparrow$	$\leftrightarrow$	$\uparrow$
Failure Risk	$\updownarrow$	$\uparrow$	$\leftrightarrow$
Lending	$\downarrow$	$\leftrightarrow$	$\downarrow$

Table 2: *Adjustment in a Downturn*

requirements are relaxed if the bank capital supply falls and tightened if productivity deteriorates. However, the extent of their responses differs: Generally, an economy with optimal regulation adjusts at both margins - risk and lending - to a shock, whereas an economy with risk-sensitive capital requirements adjusts lending only. In a capital crunch, optimal capital requirements are relaxed in order to prevent a massive contraction of the loan supply thereby tolerating a higher failure risk, whereas risk-sensitive capital requirements target a fixed risk level and are only relaxed because of a higher lending rate. Flat capital requirements are, by definition, independent of the state of the economy. A shortage of bank capital directly lowers the loan supply, which is a multiple of bank capital, and investment; through a higher equilibrium lending rate, this even makes banks safer. In this system, an adverse productivity shock only increases failure risk as lower loan demand drives down the lending rate but the loan volume is completely insensitive due to the inelastic loan supply. Despite being less valuable, the same number of projects is undertaken. These responses characterize an economy with scarce bank capital: $K < \min\left\{\bar{k}, (1-\alpha)\left(1-\frac{H(\hat{x}')}{1-\theta x_0}\right)\right\}\mu$. If the latter is available in abundant supply, a shortage of bank capital does not have any real effect but an adverse productivity shock is associated with smaller lending and, in case of flat capital requirements, higher bank risk.

3.4.3 Numerical Example

In this section, we compute the equilibrium of the model and provide a numerical example in order to illustrate the adjustment to adverse financial and real shocks. The purpose of this example is purely illustrative. The baseline calibration is $R = 1.5$, $p = 0.2$,

$\alpha = 0.55$, $\theta = 0.3$, $c = 0.1$, $K = 0.03$.[14] Furthermore, the adverse shock x is uniformly distributed on the unit interval. The expected net return μ, which equals investment in the unregulated market equilibrium, is 0.31. Hence, bank capital is clearly scarce as $K = 0.03 < 0.1395 = (1 - \alpha)\mu$. The optimal allocation is characterized by lending of 0.2710, a capital ratio of 11%, and a (gross) lending rate of 1.161. Banks fail in a recession as soon as more than 34.61% of their borrowers default which corresponds to am *ex ante* failure probability of 19.6%; social welfare (aggregate surplus) equals 0.042. We simulate two scenarios depending on whether optimal, flat or risk-sensitive capital requirement are in place: (i) a capital crunch where the supply of bank capital K falls by one third to 0.02 and (ii) an adverse productivity shock where the project return falls by ten percent to 1.35 such that the expected net return μ decreases to 0.19. The values of the baseline allocation are used to fix the flat capital requirements ($\bar{k} = 11\%$) and target risk in the risk-sensitive system ($\hat{x}' = 0.3461$). Contrary to optimal regulation, either the capital ratio or failure risk remain constant.

	Capital Requirements		
	Optimal	Flat	Risk-Sensitive
	Capital Crunch (K −33.3%)		
Capital Requirement (in pp)	-3.65	-	-2.54
Failure Risk (in pp)	+2.32	-3.36	-
Lending (in %)	-0.6	-33.32	-13.51
Welfare (in %)	-2.38	-2.38	-12.62
	Adverse Productivity Shock (R −10%)		
Capital Requirement (in pp)	+7.91	-	+3.66
Failure Risk (in pp)	-4.75	+7.51	-
Lending (in %)	-41.70	-	-24.83
Welfare (in %)	-63.81	-82.38	-66.67

Table 3: *Numerical Example*

In a capital crunch, optimal capital requirements are relaxed from 11.1 to 7.42 percent: This significantly mitigates the decline of lending (-0.6%), which would fall by one third if capital requirements were not adjusted or still by more than 12 percent if only a passive adjustment as a result of the higher lending rate occurred. However, it moderately raises the probability of a banking crisis from 19.6 to 21.9 percent. The shortage of bank capital

[14]Following Repullo and Martinez-Miera (2010), we set the liquidation value α similar to the Basel II IRB approach, which suggests a loss given default (i.e., $1 - \alpha$) of 0.45 for senior claims on corporates (foundation approach); see par. 273 in BCBS (2004).

only causes a small welfare loss under optimal and risk-sensitive capital regulation; flat capital requirements perform significantly worse. The former is remarkable because the responses of risk and lending clearly differ between the optimal and risk-sensitive system. Since an adverse productivity shock reduces loan demand and makes investment less valuable, it is optimal to exploit this and to strongly increase capital requirement from 11 to almost 19 percent in order to reduce bank risk by 4.75 percentage points. Lower loan demand and tighter regulation cause a strong decline of lending by 41.7 percent. In the risk-sensitive system, capital requirements are only passively adjusted to account for the lower lending rate and lending falls by roughly one quarter. The adjustment under flat capital requirements markedly differs: The resource allocation remains unchanged but as the lower loan demand reduces the gross lending rate from 1.16 to 1.01, financial stability is in jeopardy with banks' failure probability strongly increasing to more than 27 percent. In general, the welfare loss defined in terms of aggregate surplus[15] is higher than during a capital crunch partly because, in addition to distortions of risk or lending, all entrepreneurs are less productive. Again, optimal perform slightly better than risk-sensitive capital requirements, whereas flat capital requirements clearly exacerbate the welfare loss.

3.5 Entrepreneurial Moral Hazard

Borrowing and lending is often characterized by frictions, for instance, borrowers who are protected by limited liability and cannot be costlessly monitored by their lenders may have an incentive to allocate funds to riskier investments or to deliberately reduce effort. Adverse selection and moral hazard essentially make the lending rate a critical determinant of loan risk as shown by Stiglitz and Weiss (1981). Subsequently, we add such a credit friction to the model but stick to a formulation with private benefits in the spirit of Holmström and Tirole (1997) instead of different project returns[16]. The purpose of this extension is twofold: (i) it provides a robustness check and (ii) it allows us to study the impact of entrepreneurial moral hazard on bank capital requirements.

Suppose that the effort of an entrepreneur is critical for the project's success. More

[15] This is the reason why the relative welfare losses are so large; if welfare is defined in terms of (gross) output, they are substantially smaller.

[16] This allows separating project return and corporate governance such that the cyclical adjustment is better comparable to the baseline model.

specifically, the entrepreneur may exert effort such that the project succeeds with an *ex ante* probability $1-p_0$ or exert no effort ('shirking') such that the success probability falls to $1-p_0'$ with $p_0' = p_0 + \Delta p_0$ with $\Delta p_0 > 0$.[17] Shirking yields private benefits b for the entrepreneur but makes the project unprofitable $(1-p_0')R + p_0'\alpha - 1 < 0$. Importantly, the bank does not observe effort, which gives rise to entrepreneurial moral hazard. As a result, the lending contract needs to be incentive-compatible as to guarantee effort:

$$(1-p_0)(R-r_L) \geq (1-p_0')(R-r_L) + b \quad \Rightarrow \quad R - r_L \geq \frac{b}{\Delta p_0} \equiv \beta \tag{19}$$

Essentially, moral hazard limits the lending rate by an upper bound $R-\beta$. The parameter β is a measure of corporate governance; higher values point to a more severe agency problem. It ultimately depends on the institutional quality in a country and thus on factors like, for example, investor protection or transparency. A regulator does not observe effort either and thus needs to choose an incentive-compatible allocation. Subsequently, we focus on a binding incentive compatibility constraint, $r_L = R - \beta$, which implies that the agency problem is severe enough, and the baseline allocation would not be incentive-compatible.[18] Hence, the participation constraint is slack and the regulator's choices are subject to the binding incentive compatibility constraint:

PROGRAM 2 *The regulator maximizes welfare by choosing the capital ratio k, lending L, and the lending rate r_L*

$$\max_{k,L,r_L} [\mu - \theta(1-F(\hat{x}))c]L - \frac{L^2}{2}$$

subject to the incentive compatibility constraint of entrepreneurs, $r_L = R - \beta$, and the capital availability constraint, $K \geq kL$.

Solving this program yields the second-best equilibrium allocation:

[17] Shirking increases the idiosyncratic project risk by Δp such that $\Delta p_0 = (1-\theta x_0)\Delta p$ and $p_0' = p_0 + \Delta p_0 = p + \Delta p + \theta x_0[1-(p+\Delta p)]$.

[18] However, moral hazard must not be too severe, $\beta \leq \mu$, as banks would otherwise earn negative expected profits or their shareholders a negative return. This follows from $\pi^B = [(1-p)r_L + p\alpha - 1 - (\gamma-1)k]L = [\mu - \beta - (\gamma-1)k]L \geq 0$, using $r_L = R - \beta$. $\beta > \mu$ would lead to full credit rationing. Whenever $K \geq K_0$, the lending rate is small such that the IC is slack; $r_L = (1-p\alpha)/(1-p) < R - \beta$. Hence, entrepreneurial moral hazard may only arise if scarce bank capital is required to internalize the social cost of bank failure.

LEMMA 4 *Bank lending and failure threshold are determined by the system*

$$J^1(L,\hat{x}') = \mu - L' - c\theta[1-F(\hat{x}')] - \frac{c\theta f(\hat{x}')}{1-\theta+\theta F(\hat{x}')}\left[\frac{1-\alpha}{(1-p)(R-\beta-\alpha)} - H(\hat{x}')\right] = 0 \tag{20}$$

$$J^2(L,\hat{x}') = K - [1-\alpha-(R-\beta-\alpha)(1-p)H(\hat{x}')]\,L = 0 \tag{21}$$

Lending increases in the supply of bank capital K and entrepreneurs' productivity R but decreases in the corporate governance parameter β. The failure threshold increases in the supply of bank capital but the sensitivity with respect to productivity and corporate governance can be of either sign.

Proof: See Appendix A.1.

The first-order condition for loans and the capital availability constraint are similar to the baseline model apart from the fixed lending rate. The equilibrium allocation involves capital regulation:

PROPOSITION 3 *Banks are subject to optimal capital requirements*

$$k'(r_L,\hat{x}') = 1-\alpha-(r_L-\alpha)(1-p)H(\hat{x}')$$

with $r_L = R-\beta$ that increase in the supply of bank capital, $\frac{\partial k'(r_L,\hat{x}')}{\partial K} > 0$, and the corporate governance parameter, $\frac{\partial k'(r_L,\hat{x}')}{\partial \beta} > 0$, and decrease in entrepreneurs' productivity, $\frac{\partial k'(r_L,\hat{x}')}{\partial R} < 0$.

Proof: See Appendix A.1.

This proposition has two main implications: First, moral hazard leads to an artificially low lending rate as higher rates would destroy incentives, an effect reinforced by a poor corporate governance. Since banks earn a smaller interest income and are *ceteris paribus* riskier, the optimal capital ratio increases such that poor governance of firms and entrepreneurial projects requires tighter capital regulation. In the presence of such frictions, optimal regulation thus depends on a country's institutional quality. Second, the adjustment of capital requirements to economic shocks is qualitatively similar to the baseline model without moral hazard. However, the mechanisms differ: Since the lending rate is fixed, some of the equilibrium effects that are relevant in the standard model disappear. Capital requirements are relaxed in capital crunch only because tolerating a higher bank risk is optimal and not due to a declining lending rate. The tightening in case of an adverse productivity shock, in contrast, still results from the two effects described above: The lending rate decreases to preserve the incentive of entrepreneurs, which, in turn, requires a higher capital ratio to keep failure risk constant. In addition, the regulator may adjust the latter in either way but even if a higher risk level is tolerated, the first, positive effect prevails.

4 Conclusion

This paper provides a normative analysis of the cyclical adjustment of capital regulation, the purpose of which is to internalize the social cost of a systemic banking crisis. The latter originates from the real sector, which may suffer from an adverse macroeconomic shock in a recession, such that correlated defaults lead to bank failure. As long as bank capital is scarce, optimal regulation balances the trade-off between financial stability and the investment capacity of the real sector. Two channels link optimal capital requirements to the state of the economy: target risk and the equilibrium lending rate. The latter, which, contrary to related models like Kashyap and Stein (2004) and Repullo (2013), is endogenized by a full-fledged model of the real sector, plays an important role as a *de facto* substitute for bank capital on the 'risk front' and thus strongly influences the optimal regulatory adjustment. The main finding is that there are striking differences depending on the driving force of an economic downturn: If the supply of bank capital falls, banks face difficulties to raise equity such that they may reduce the loan supply. Therefore, capital requirements should be less strict: On the one hand, a smaller loan supply raises the lending rate, which has a stabilizing effect by making banks safer due a higher interest income. On the other hand, tolerating a higher bank failure risk is optimal in order to prevent a sharp fall of lending and investment. In a downturn primarily characterized by poor investment prospects and a low loan demand of entrepreneurs, capital requirements should be tighter: Declining lending rates undermine a bank's resilience such that the risk level is only preserved at a higher capital ratio. More importantly, the number of attractive investments is small; such a situation even allows a regulator to further improve financial stability by tighter capital requirements at relatively small economic cost. Consequently, regulation should be tough whenever bank capital is easily available

and only few investments are promising but can be relaxed whenever equity is scarce and investment prospects are good. Contrary to flat or risk-sensitive capital requirements, optimal regulation allows the economy to adjust to shocks at two different margins - lending and risk - whereas one of them is *de facto* fixed in the two alternative systems, which exacerbates the welfare losses in a downturn. The main findings also result in the presence of entrepreneurial moral hazard, which makes optimal regulation sensitive to additional factor such as corporate governance and institutional quality. Eventually, the supply of bank capital can be large enough to make banks safe without harming investment. In this case, capital requirements are insensitive to economic shocks.
How do these findings compare with the countercyclical buffer envisaged in Basel III? It is of course difficult to interpret such a buffer, which is essentially a dynamic concept as the buffer is accumulated during periods of excess credit growth and effectively relaxed in a downturn, in the context of our static framework. One implication of our results is that it makes a difference whether credit growth is mainly supply- or demand-driven as capital requirements should indeed be tightened in the first but relaxed in the second case to accommodate the higher demand. Hence, the regulator should identify the precise source of credit growth and activate the buffer primarily in case attractive funding conditions of banks let their loan supply growing rapidly. In line with the focus on the real sector, our analysis of course mainly concerns corporate and business loans that finance productive real investments; credit growth driven by a strong demand for mortgages, however, does not necessarily imply that relaxing capital requirements is optimal because it may rather reflect a real estate bubble instead of more productive and valuable investments.

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A Appendix

A.1 Proofs

Proof of Lemma 1 The central condition follows from combining (2) and (5): Using integration by parts, the participation constraint can be written as:

$$\theta\left[\alpha - [(1-p)(1-\hat{x})r_L + (p+\hat{x}(1-p))]F(\hat{x}) + (r_L-\alpha)(1-p)\int_{\hat{x}}^{1} F(x)dx\right]$$
$$+[1-\theta+\theta F(\hat{x})]r(1-k) = 1-k$$

The definition of the failure threshold implies $(1-p)(1-\hat{x})r_L + (p+\hat{x}(1-p)) = r(1-k)$, which can be used to eliminate $r(1-k)$ such that

$$\theta\left[\alpha + (r_L-\alpha)(1-p)\int_{\hat{x}}^{1} F(x)dx\right] + (1-\theta)[(1-p)(1-\hat{x})r_L + (p+\hat{x}(1-p))] = 1-k$$

which can be rearranged to get (6). Eventually, differentiating this condition yields the sensitivities

$$\frac{\partial \hat{x}}{\partial k} = \frac{1}{[1-\theta+\theta F(\hat{x})](1-p)(r_L-\alpha)} > 0$$
$$\frac{\partial \hat{x}}{\partial r_L} = \frac{H(\hat{x})}{[1-\theta+\theta F(\hat{x})](r_L-\alpha)} > 0$$
$$\frac{\partial \hat{x}}{\partial \alpha} = \frac{1-(1-p)H(\hat{x})}{[1-\theta+\theta F(\hat{x})](1-p)(r_L-\alpha)} > 0$$

where the latter uses $H(\hat{x}) \leq 1-\theta x_0$. *Q.E.D.*

Proof of Proposition 1 Program 1 can be stated as a Lagrangian

$$\mathcal{L}(k, L, \lambda_1, \lambda_2) = [\mu - \theta(1-F(\hat{x}))c]L - \frac{L^2}{2} + \lambda_1[K - kL] + \lambda_2[(1-p_0)(R-_L) - L]$$

which uses $L = \hat{u}$ in equilibrium. The corresponding first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial k} = \frac{c\theta f(\hat{x})L}{(1-p)(r_L-\alpha)[1-\theta+\theta F(\hat{x})]} - \lambda_1 L = 0$$
$$\frac{\partial \mathcal{L}}{\partial L} = \mu - (1-F(\hat{x}))c - L - \lambda_1 k - \lambda_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial r_L} = \frac{c\theta f(\hat{x})[1-\theta-H(\hat{x})]L}{(r_L-\alpha)[1-\theta+\theta F(\hat{x})]} - \lambda_2(1-p_0) = 0$$

as well as participation and capital availability constraint. The first condition is equivalent to equation (12) and implies a positive capital ratio. Thus, one can rearrange (6) to express the capital ratio as a function of the risk level: $k = 1-\alpha-(1-p)(r_L-\alpha)H(\hat{x})$. It is more convenient and intuitive to substitute this expression for k in the above conditions and to treat $\hat{x}$ as the unknown.

The problem can be reduced to the system (14) - (15) with two equations and two unknowns: L and $\hat{x}$. Substituting for the Lagrange multipliers, the capital ratio, and the lending rate in $\frac{\partial \mathcal{L}}{\partial L} = 0$ yields equation $J^1_L(\hat{x}, L) = 0$. The second equation, $J^2_L 1(\hat{x}, L) = 0$, is the capital availability constraint where we substitute for k using $r_L = R - \frac{L}{1-p}$. The system has the solutions $\hat{x} = 1$ and $L = \mu$ as soon as $K \geq K_0$, which allows for $k = 1-\alpha$. Totally differentiating yields the Jacobian

$$J = \begin{bmatrix} J^1_L & J^1_{\hat{x}} \\ J^2_L & J^2_{\hat{x}} \end{bmatrix}$$

where $J^j_i = \frac{\partial J^j}{\partial i}$ with $i = \{L, \hat{x}\}$ and $j = \{1, 2\}$ and

$$\begin{aligned}
J^1_L &= -\left[1 + \frac{c\theta f(\hat{x})\left[1-\alpha+(1-p)(R-\alpha)H(\hat{x})\right]}{[1-\theta+\theta F(\hat{x})](1-p_0)(1-p)(r_L-\alpha)^2}\right] < 0 \\
J^1_{\hat{x}} &= \frac{c\theta f(\hat{x})L}{(r_L-\alpha)(1-p_0)} + \frac{c\theta\left[\theta f(\hat{x})^2 - (1-\theta+\theta F(\hat{x}))f'(\hat{x})\right]\left[k + \frac{(1-p)H(\hat{x})L}{1-p_0}\right]}{[1-\theta+\theta F(\hat{x})]^2(r_L-\alpha)} \geq 0 \\
J^2_L &= -\left[k + \frac{(1-p)H(\hat{x})L}{1-p_0}\right] < 0 \\
J^2_{\hat{x}} &= -(1-p)(r_L-\alpha)[1-\theta+\theta F(\hat{x})]L < 0
\end{aligned}$$

The Jacobian determinant is positive:

$$\nabla = J^1_L J^2_{\hat{x}} - J^2_L J^1_{\hat{x}} > 0$$

The comparative statics are obtained using Cramer's rule. A larger supply of bank capital increases lending and the failure threshold:

$$\frac{\partial L}{\partial K} = \frac{J^1_{\hat{x}}}{\nabla} \geq 0, \quad \frac{\partial \hat{x}}{\partial K} = -\frac{J^1_L}{\nabla} > 0$$

A higher productivity of entrepreneurs increases lending and but the response of the failure threshold is ambiguous:

$$\frac{\partial L}{\partial R} = \frac{-J_R^1 J_{\hat{x}}^2 + J_R^2 J_{\hat{x}}^1}{\nabla} > 0, \quad \frac{\partial \hat{x}}{\partial R} = \frac{-J_L^1 J_R^2 + J_L^2 J_R^1}{\nabla} = -\frac{kJ_R^1 - \frac{c\theta(1-p)f(\hat{x})H(\hat{x})^2 L}{(1-p_0)(r_L-\alpha)[1-\theta+\theta F(\hat{x})]}}{\nabla}$$

where J_R^k for $k = \{1, 2\}$ are given by

$$J_R^1 = 1 - p_0 + \frac{c\theta f(\hat{x})}{1-\theta+\theta F(\hat{x})} \frac{1-\alpha+\frac{(1-p)H(\hat{x})L}{1-p_0}}{(1-p)(r_L-\alpha)^2} > 0, \quad J_R^2 = (1-p)H(\hat{x})L > 0$$

Note that $\frac{\partial \hat{x}}{\partial R}$ is usually negative unless the equilibrium capital ratio is very low and, at the same time, the social cost of bank failure is very high. $Q.E.D.$

Proof of Corollary 2 Substituting the capital ratio (17) into the definition of the failure threshold (6) shows that bank risk is, by construction, optimal: $\hat{x} = \hat{x}^*$. Each bank chooses loans and capital structure as to maximize its expected profit π^B defined in (2) subject to the regulatory constraint $k \geq k^*$. The Lagrangian is

$$\mathcal{L}(k, L, \eta) = [(1-p_0)r_L + p_p\alpha - (1-k) - \gamma k]L + \eta[k - k^*]$$

where η is the Lagrange multiplier of the regulatory constraint. The corresponding first-order conditions are:

$$(1-p)r_L + p\alpha - 1 - (\gamma-1)k = 0, \quad -(\gamma-1) + \eta = 0, \quad \eta(k-k^*) = 0$$

Substituting $r_L = R - \frac{L}{1-p}$ using loan market clearing and the definition of the marginal entrepreneur, yields bank lending:

$$L = \mu - (\gamma-1)k$$

If $K < K_0$, capital requirements are $k^* < 1-\alpha$. First, we show that the regulatory constraint binds: Suppose banks chose a higher capital ratio $k > k^*$, dividing market clearing, $K = kL$, by k would yield $K/k = L = \mu - (\gamma-1)k$. The left-hand side falls short of optimal lending such that $L < L^* = K/k^*$, which is smaller than μ according

to proposition 1. From above, we have $\gamma > 1$ and $\eta > 0$, which is incompatible with complementary slackness. Consequently, capital requirements are binding: $k = k^*$. By substituting $k^* = K/L^*$ into the market clearing condition, $K = k^*L$, one observes that bank provide the optimal amount of loans, $L = L^*$.
Whenever $K \geq K_0$, capital requirements are $k^* = 1 - \alpha$. Banks choose $k \in [1-\alpha, K/\mu]$ such that from market clearing, $K \geq k[\mu - (\gamma - 1)k]$, w $\gamma = 1$ (which implies $\eta = 0$ and allows for a possibly non-binding regulatory constraint) and banks choose optimal lending $L = \mu$. An even higher capital ratio, $k > K/\mu$ would lead to an inefficiently small amount of loans but is ruled out by complementary slackness. *Q.E.D.*

Proof of Proposition 2 Using the definition of capital requirements, (17), the sensitivity w.r.t. the bank capital supply K is

$$\frac{\partial k^*(r_L, x^*)}{\partial K} = -(1-p)H(\hat{x})\frac{\partial r_L}{\partial K} + (r_L - \alpha)(1-p)[1 - \theta + \theta F(\hat{x}^*)]\frac{\partial \hat{x}^*}{\partial K} > 0$$

where $\frac{\partial r_L}{\partial K} = -\frac{1}{1-p}\frac{\partial L^*}{\partial K} < 0$; the positive sign follows from $\frac{\partial L}{\partial K} > 0$ and $\frac{\partial \hat{x}^*}{\partial K} > 0$ given by proposition 2. Similarly, one can derive the sensitivity w.r.t. productivity R

$$\frac{\partial k^*(r_L, x^*)}{\partial R} = -(1-p)H(\hat{x})\frac{\partial r_L}{\partial R} + (r_L - \alpha)(1-p)[1 - \theta + \theta F(\hat{x}^*)]\frac{\partial \hat{x}^*}{\partial R}$$

where $\frac{\partial r_L}{\partial R} = 1 - \frac{1}{1-p_0}\frac{\partial L^*}{\partial R} = \frac{(1-p_0)\nabla + J_R^1 J_{\hat{x}}^2 - J_R^2 J_{\hat{x}}^1}{(1-p_0)\nabla} > 0$. However, the sign of $\frac{\partial \hat{x}^*}{\partial K}$ is ambiguous. Using the sensitivities from the proof of proposition 2, one can show that optimal capital requirements decrease in productivity:

$$\begin{aligned}\frac{\partial k^*}{\partial R} &= -\frac{1-p}{\nabla}\left[H(\hat{x})\frac{(1-p_0)\nabla + J_R^1 J_{\hat{x}}^2 - J_R^2 J_{\hat{x}}^1}{1-p_0} + k(r_L - \alpha)[1 - \theta + \theta F(\hat{x})]J_R^1 - \frac{c\theta(1-p)f(\hat{x})H(\hat{x})^2 L}{(1-p_0)}\right] \\ &= -\frac{(1-p)k}{\nabla}\left[H(\hat{x})J_{\hat{x}}^1 + (r_L - \alpha)[1 - \theta + \theta F(\hat{x})]J_R^1\right] < 0\end{aligned}$$

Alternatively, one can derive this result using the capital availability constraint: Since the supply is fixed and lending increases in R, the capital ratio necessarily falls. Once $K \geq K_0$, the optimal capital requirements $k = 1 - \alpha$ are clearly independent of both entrepreneurs' productivity and the bank capital supply. *Q.E.D.*

Proof of Lemma 4 If the incentive compatibility constraint binds such that $r_L = R - \beta$, the equilibrium conditions are (20), the first-order condition w.r.t L substituting for the Lagrange multiplier $\lambda = \frac{cf(\hat{x})}{1-F(\hat{x})}\frac{\partial \hat{x}}{\partial k}$, and (21), the capital availability constraint substituting for k and r_L. Differentiating (20) - (21) yields the Jacobian with:

$$J_L^1 = -1 < 0, \quad J_{\hat{x}}^1 = \frac{c\theta \left[\theta f(\hat{x})^2 - f'(\hat{x})(1-\theta+\theta F(\hat{x}))\right] k}{(r_L-\alpha)(1-p)(1-\theta+\theta F(\hat{x}))^2} \geq 0$$
$$J_L^2 = -k < 0, \quad J_{\hat{x}}^2 = -(r_L-\alpha)(1-p)(1-\theta+\theta F(\hat{x}))L < 0$$

The Jacobian determinant is positive: $\nabla = J_L^1 J_{\hat{x}}^2 - J_L^2 J_{\hat{x}}^1 > 0$. Applying Cramer's rule, we find that larger supply of bank capital increases lending and the failure threshold

$$\frac{\partial L}{\partial K} = \frac{J_{\hat{x}}^1}{\nabla} > 0, \quad \frac{\partial \hat{x}}{\partial K} = -\frac{J_L^1}{\nabla} > 0$$

and that higher productivity increases lending but has an ambiguous effect on the failure threshold

$$\frac{\partial L}{\partial R} = \frac{-J_R^1 J_{\hat{x}}^2 + J_R^2 J_{\hat{x}}^1}{\nabla} > 0, \quad \frac{\partial \hat{x}}{\partial R} = \frac{J_R^2 - kJ_R^1}{\nabla}$$

Note that J_R^i for $i = \{1, 2\}$ are

$$J_R^1 = 1 -_0 + \frac{c\theta f(\hat{x})}{1-\theta+\theta F(\hat{x})} \frac{1-\alpha}{(r_L-\alpha)^2(1-p)} > 0, \quad J_R^2 = (1-p)H(\hat{x})L > 0$$

Poor corporate governance of entrepreneurs lowers lending but has an ambiguous impact on the failure threshold

$$\frac{\partial L}{\partial \beta} = \frac{-J_\beta^1 J_{\hat{x}}^2 + J_\beta^2 J_{\hat{x}}^1}{\nabla} < 0, \quad \frac{\partial \hat{x}}{\partial \beta} = \frac{J_\beta^2 - kJ_\beta^1}{\nabla}$$

with J_β^j for $j = \{1, 2\}$.

$$J_\beta^1 = -\frac{c\theta f(\hat{x})}{1-\theta+(\hat{x})} \frac{1-\alpha}{(r_L-\alpha)^2(1-p)} < 0, \quad J_\beta^2 = -(1-p)H(\hat{x})L < 0$$

Q.E.D.

Proof of Proposition 3 The positive response of capital requirements $k' = 1 - \alpha - (R - \beta - \alpha)H(\hat{x})$ to a larger supply of bank capital is due to a higher failure threshold

$\frac{\partial \hat{x}'}{\partial K} > 0$. The sensitivity w.r.t. productivity follows from

$$\begin{aligned}\frac{\partial k'}{\partial R} &= -(1-p)H(\hat{x}) + (r_L - \alpha)(1-p)[1-\theta+\theta F(\hat{x})]\frac{\partial \hat{x}}{\partial R} \\ &= -\frac{(1-p)k\left[H(\hat{x})J^1_{\hat{x}} + (r_L-\alpha)[1-\theta+\theta F(\hat{x})]J^1_R\right]}{\nabla} < 0\end{aligned}$$

and is negative. Similarly, the sensitivity w.r.t. the corporate governance parameter is

$$\begin{aligned}\frac{\partial k'}{\partial \beta} &= (1-p)H(\hat{x}) + (r_L - \alpha)(1-p)[1-\theta+\theta F(\hat{x})]\frac{\partial \hat{x}}{\partial \beta} \\ &= \frac{(1-p)k\left[H(\hat{x})J^1_{\hat{x}} - (r_L-\alpha)[1-\theta+\theta F(\hat{x})]J^1_\beta\right]}{\nabla} > 0\end{aligned}$$

and positive. These two effects are also implied by the response of bank lending in the presence of a fixed bank capital supply. *Q.E.D.*